Simplified model of underwater electrical discharge

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A model of the underwater discharge with initiating wire is presented. The model reveals the nature of similarity parameters which have been phenomenologically introduced in earlier experimental research in order to predict behavior of different discharges. It is shown that these parameters naturally appear as a result of the normalization of differential equations, which determines the process of underwater wire initiated discharge. In these equations the energy conservation law for wire material evaporation and the dependence of plasma conductivity on the energy dissipated in the discharge are implied to calculate the time varying resistance of the discharge gap. The comparison of calculations with the experimental results shows that good agreement is achieved when modification of these parameters is introduced. These new similarity parameters are functions of the original similarity parameters, hence the law of the similarity of underwater electrical discharge is preserved.

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I. INTRODUCTION

The subject of underwater electrical discharge (UED) has been under intense theoretical and experimental investigation for more than 50 years because of many important technical applications and sophisticated physical phenomena involved in this process [1–6]. At an applied voltage of $\leqslant$50 kV and a gap of several centimeters, the discharge is initiated by an electrical explosion of conducting wire which shorts this gap. In the case of small wire diameters ($<$0.1 mm), this wire serves only as an igniter and the main discharge occurs in the ionized water vapor. In the case of a thick wire ($>$0.1 mm), the discharge occurs mainly in the ionized vapor of the electrically evaporated wire material. In both cases, UED is accompanied by phase transitions of the wire material and water, namely, melting, evaporation, and plasma formation. This type of discharge depends on many parameters that include wire properties, such as wire diameter, length and material, the characteristics of the electrical circuit, and the properties of the background medium. Uncertainty in time-dependent parameters, such as specific resistance, thermal conductivity, density, and temperature of the wire and water during phase transitions, makes it difficult to provide a self-consistent description of UED [5–7].

However, in spite of the complexity of phenomena involved in UED, numerous experimental investigations have shown that this type of the discharge obeys a certain similarity law [5,6,8–10]. It has been demonstrated that identical dimensionless current and voltage waveforms are obtained in different discharges when three dimensionless combinations of various discharge setup parameters, called the "similarity parameters," are identical. Here let us note that the existence of the similarity has been validated experimentally in microsecond UED for current amplitudes of 10^4–10^6 A with different types of gap shorting wires. Nevertheless, the empirical approach does not offer a physical explanation for the existence of the similarity of discharges.

In this paper we present a model which provides a physical insight into the process of the underwater wire initiated discharge. It is assumed that main processes governing UED are the wire evaporation due to Ohmic heating and plasma channel formation in water due to dissipated energy. As a natural outcome of these simple assumptions, all the similarity parameters which were "intelligently guessed" [6,9,10] and validated by numerous experiments emerge. A comparison of numerically and experimentally obtained data is presented and the difference between them is discussed.

II. A MODEL OF ELECTRICAL WIRE EVAPORATION

Let us first consider an electrical explosion of a wire by the use of capacitor discharge. A simplified electrical scheme is composed of the following parts connected in series: a capacitor C charged to a potential φ_0, an inductance L, a switch, and a discharge gap shorted by a wire with initial resistance R_{0m}, cross-sectional area S_0, length l, and initial specific conductivity and density σ_0 and ρ_0, respectively. When the switch is closed, the discharge proceeds according to the well-known equation for this electrical circuit:

$$LC(d^2\varphi/dt^2)+R_m(t)C(d\varphi/dt)+\varphi=0. \tag{1}$$

Here $R_m(t)$ is an unknown time-varying resistance of the current-carrying wire. The discharge current causes intense heating, melting, evaporation and ionization of the wire material. In our model we assume a decrease of the wire cross-sectional area due to radial uniform evaporation of the wire boundary. In this case, by neglecting radiation energy losses and taking λ [(in units of J/kg)] as the specific energy required for transformation of the wire material into the plasma, one can write

$$\lambda\frac{dM}{dt}=-R_m(t)C^2\left(\frac{d\varphi}{dt}\right)^2 \tag{2}$$

where $M(t)$ is the mass of the wire. Thus, the process of wire transformation into the plasma is determined only by the thermal energy absorbed in the conducting wire. In the case of a cross-section uniform conductivity, the wire resistance is expressed as

$$R_m(t)=\frac{l}{\sigma(t)S(t)}. \tag{3}$$

Here $\sigma(t)$ is specific wire conductivity which generally depends on the wire temperature as well as the density $\rho(t)$ and $S(t)$ is the wire cross sectional area. However, in our simplified model we assume $\sigma=\sigma_0$ and $\rho=\rho_0$. Using these assumptions and Eq. (3) one can write Eq. (2) in the following form:

$$\frac{dR_m}{dt}=R_m\left[\frac{C^2}{\lambda\sigma_0\rho_0 S^2(t)}\right]\left(\frac{d\varphi}{dt}\right)^2. \tag{4}$$

Next, let us normalize Eq. (4) and introduce the following dimensionless variables: dimensionless voltage $u(t)=\varphi(t)/\varphi_0$, dimensionless cross sectional area $s(t)=S(t)/S_0$, where S_0 is the initial wire cross sectional area, dimensionless resistance $r_m(t)=R_m(t)/R_{0m}$, dimensionless time $\tau=t/\sqrt{LC}$, and the characteristic wave impedance of the electrical circuit $Z=\sqrt{L/C}$. From these definitions it follows that $r_m(\tau)=1/s(\tau)$.

Now Eqs. (1) and (4) can be written in dimensionless form as

$$\frac{d^2u}{d\tau^2}+P_1 r_m\frac{du}{d\tau}+u=0, \tag{5a}$$

$$\frac{dr_m}{d\tau}=P_2 r_m^3\left(\frac{du}{d\tau}\right)^2. \tag{5b}$$

The initial conditions of these differential equations at $\tau=0$ read as follows: $u=r_m=1$ and $du/d\tau=0$. In these equations appear two dimensionless parameters:

$$P_1\equiv\frac{l}{\sigma_0 S_0}\frac{1}{Z} \tag{6}$$

and

$$P_2\equiv\left(\frac{C}{Z\lambda\sigma_0\rho_0}\right)\left(\frac{\varphi_0^2}{S_0^2}\right). \tag{7}$$

These parameters are identical to the first two out of the three similarity parameters which have been obtained empirically [6]. The physical meaning of these similarity parameters follows from an examination of Eqs. (6) and (7). The first similarity parameter, which can be written as

$$P_1=\frac{R_{0m}}{Z},$$

gives the ratio between the initial wire resistance and the electrical circuit wave impedance.

The second similarity parameter can be written as

$$P_2=\left(\frac{C\varphi_0^2/2}{\lambda M_0}\right)\left(\frac{2R_{0m}}{Z}\right)=\left(\frac{W_E}{W_m}\right)\left(\frac{2R_{0m}}{Z}\right)$$

where W_E is the electrically stored energy and W_m is the energy required for complete evaporation of the wire. One can see that this parameter determines the rate of wire evaporation [see Eq. (5b)]. Indeed, the increase (decrease) of R_{0m} or decrease (increase) of W_m relatively to Z and W_E respectively, increases (decreases) the evaporation rate. The same physical meaning of P_1 and P_2 can be assigned to the similarity parameters which have been ''intelligently guessed,'' but here it is explicitly shown what processes are responsible for the appearance of these parameters. Let us note that formal dimension analysis shows that these are not the only possible dimensionless combinations that can be constructed from the discharge circuit parameters [8]. Therefore, a simple dimension consideration without the consideration of physical processes cannot give these parameters.

Obviously, given a set of two numbers P_1 and P_2, one uniquely determines the dependence of normalized voltage $u(\tau)$ and normalized current $du(\tau)/d\tau$. However, the same numerical values of P_1 and P_2 can be obtained for different experimental setups. The latter means that the same normalized current and voltage waveforms will be obtained for these different setups. This is the essence of similarity.

Let us note that Eq. (5), where parameters P_1 and P_2 have naturally appeared, accounts only for the simplest process responsible for the wire explosion. Namely, an increase of wire resistance results from the decrease of the wire diameter due to the uniform evaporation of the wire boundary. Nevertheless, the similarity of different discharges with the same P_1 and P_2 has been firmly validated by experiments. The latter surprisingly indicates that the considered simple mechanism of the wire resistance dynamics is the main process that governs this type of the discharge. As will be shown later, in the case of relatively thick wire where other processes become significant, a straightforward application of these parameters in Eq. (5) does not give appropriate agreement of the experimentally obtained and calculated current waveforms. Therefore, the original parameters P_1 and P_2 introduced in Eqs. (6) and (7) should be modified in order to obtain satisfactory agreement with experimentally obtained data. It will be shown that these modified parameters are uniquely determined by P_1 and P_2, such that the similarity is essentially preserved.

The decrease of the wire diameter also leads to the increase of wire inductance. Therefore, it could be important to consider an inductive voltage

$$\varphi_L=-C\frac{d\varphi}{dt}\frac{dL}{dt}.$$

In case of a wire, its dimensionless inductance is given by

$$L(\tau)\propto\ln[r_m(\tau)].$$

This means that the inductive resistance

$$r_L\propto\frac{d[\ln(r_m)]}{d\tau}=\frac{1}{r_m}\frac{dr_m}{d\tau}$$

does not grow faster than the resistance r_m even in the vicinity of the time of complete wire evaporation. This can be easily shown by analyzing the asymptotic behavior of r_m near the explosion point. For this end we seek a solution in the form

$$r_m(\tau)=a(\tau_0-\tau)^{-n},$$

where τ_0 is the time of the wire explosion and a is the constant to be determined. Substituting this solution into Eq. (5), one obtains $a=1/(2P_1)$ and $n=1$. Thus, both r_m and r_L behave according to the same law near τ_0. Therefore, one can neglect inductive resistance also during the stage of wire explosion.

III. ELECTRICAL DISCHARGE IN WATER

Now, let us consider electrical discharge in water. This type of electrical discharge was studied by Okun [9]. Further, in studies carried out by Krivitskii [6] and Krivitskii and Sholom [10], it was shown that the resistance of the discharge water channel $R_p(t)$ could be described by an empirical expression,

$$R_p(t)=Al^2(\sigma-1)/pV. \tag{8}$$

Here l is the length of the discharge channel, γ is the adiabatic constant, pV is the work of the pressure p required for the channel expansion to volume V, and A is the spark constant, which varies in the range $A=(0.25\text{–}2.5)\times10^5\ \mathrm{V^2\,s/m^2}$. We note that the dependence in Eq. (8) was validated experimentally in a microsecond time scale of UED. It is reasonable to consider that the work pV is proportional to the thermal energy W_p delivered to the discharge channel. In this case one can write Eq. (8) in the form of a differential equation for $R_p(t)$:

$$\frac{dR_p}{dt}=-\frac{R_p^3C^2}{B}\left(\frac{d\varphi}{dt}\right)^2. \tag{9}$$

Here B is the constant defined as $B=Al^2(\gamma-1)$. The dimensionless resistance $r_p(t)$ of the discharge channel is defined as $r_p(t)=R_p(t)/R_{0p}$, where R_{0p} is some typical plasma resistance which will be determined later. In this case one can rewrite Eq. (9) as

$$\frac{dr_p}{d\tau}=-\left[\frac{(R_{0p}C\varphi_0)^2}{B\sqrt{LC}}\right]\times r_p^3\left[\frac{du}{d\tau}\right]^2. \tag{10}$$

In this equation $r_p(\tau)$ and $u(\tau)$ vary in the range of 0–1. In order to determine the value of the expression in the parentheses in Eq. (10) let us rearrange this expression as

$$\frac{(R_{0p}C\varphi_0)^2}{B\sqrt{LC}}=\left(\frac{1}{2}C\varphi_0^2\right)\frac{R_{0p}}{B}\,\frac{2R_{0p}C}{\sqrt{LC}}=\frac{W_E}{W_p}\,\frac{2R_{0p}C}{\sqrt{LC}}.$$

One can see that setting the value of this expression to unity corresponds to the requirement of aperiodical electrical discharge with 100% electrical energy absorption in the discharge channel. Since we are interested in analyzing this type of discharge we set the value of the last expression to 1 and determine the typical plasma resistance R_{0p} as

$$R_{0p}\equiv\sqrt{\frac{BZ}{C\varphi_0^2}}. \tag{11}$$

Now, one can rewrite Eq. (10) in a dimensionless form,

$$\frac{dr_p}{d\tau}=-r_p^3\left[\frac{du}{d\tau}\right]^2 \tag{12a}$$

and the dimensionless equation of electric circuit becomes

$$\frac{d^2u}{d\tau^2}+P_3r_p\left(\frac{du}{d\tau}\right)+u=0. \tag{12b}$$

In the last equation a new similarity parameter P_3 appears,

$$P_3\equiv R_{0p}/Z. \tag{13}$$

This similarity parameter is identical to the third similarity parameter introduced by Krivitskii [6] and Krivitskii and Sholom [10]. From Eq. (12b) one can see that P_3 is in essence similar to P_1, however, its nature is different. It was shown that P_3 appears as a result of an assumption that the plasma channel conductivity is proportional to the energy dissipated in the channel. Experimental validation of the similarity of the plasma stage of discharges with the same P_3 indicates that this is the main process which determines the evolution of plasma channel resistance.

IV. TWO-STAGE ELECTRICAL DISCHARGE

Now let us consider UED which occurs simultaneously through both discharge channels, i.e., through the evaporating wire and plasma channels in water. This discharge can be described by connecting resistances of the wire and plasma channels in parallel. In this case the combined equation of the electric circuit can be written as

$$\frac{d^2u}{d\tau^2}+P_1\left[\frac{r_mr_p}{r_p+P_4r_m}\right]\left(\frac{du}{d\tau}\right)+u=0. \tag{14}$$

Here we introduce an additional dimensionless parameter

$$P_4\equiv R_{0m}/R_{0p}.$$

In fact, P_4 is a combination of two previously obtained dimensionless parameters P_1 and P_3 since it can be shown that $P_4=P_1/P_3$. This parameter determines the ratio between the initial wire resistance and the typical resistance of the water plasma channel. Dimensionless resistances r_m and r_p are determined by the equations similar to Eqs. (5b) and (12b),

$$\frac{dr_m}{d\tau}=P_2r_m^3\left(\frac{dU}{d\tau}\right)^2\left(\frac{r_p}{r_p+P_4r_m}\right)^2, \tag{15a}$$

$$\frac{dr_p}{d\tau}=-r_p^3\left(\frac{dU}{d\tau}\right)^2\left(\frac{P_4r_m}{r_p+P_4r_m}\right)^2. \tag{15b}$$

Here the expression in the second set of second parentheses accounts for the current distribution in parallel resistances.

Let us note that $P_4 \to 0$ corresponds to the case when all the current flows through the wire and $P_4 \to \infty$ corresponds to the discharge through the plasma channel in water. Equations (14) and (15) can be solved numerically with the following initial conditions: $r_m = 1$, $r_p = \infty$, $u = 1$, $du/d\tau = 0$. Let us note that in this model the energy which was dissipated during the wire evaporation is not accounted for in the resistance of the plasma channel.

Now, let us reconsider the discharge current flowing through two parallel channels, i.e., through the wire and through the plasma channel which is forming around the wire. The resistance of wire grows due to the decrease of its diameter. This process occurs due to the Joule heating of wire by the current which leads to the wire evaporation. Resistance of the plasma channel is determined by the energy absorbed in this channel. Therefore, a decrease of the resistance of the plasma channel is also due to the energy flux realized from the evaporating wire. Thus, one can suppose that the resistance of the plasma channel is determined by the total energy dissipated in the discharge while neglecting the energy dissipated by the radiation with a long mean free path in water. In addition, since dimensionless resistances r_m and r_p vary in the range 0–∞ it is reasonable to consider the corresponding dimensionless conductivities $y_m = 1/r_m$ and $y_p = 1/r_p$ which vary in the range 0–1. Taking into account two previous remarks, the full system of self-consistent equations which determine the process of underwater wire explosion becomes

$$\frac{d^2u}{d\tau^2} + \frac{P_1}{y_m + P_4 y_p}\frac{du}{d\tau} + u = 0, \tag{16a}$$

$$\frac{dy_m}{d\tau} = -\frac{P_2 y_m}{(y_m + P_4 y_p)^2}\left(\frac{du}{d\tau}\right)^2, \tag{16b}$$

$$\frac{dy_p}{d\tau} = \frac{P_4}{y_m + P_4 y_p}\left(\frac{du}{d\tau}\right)^2. \tag{16c}$$

V. NUMERICAL CALCULATION AND COMPARISON WITH EXPERIMENT

The intention of this work was to determine the physical origins of similarity parameters that originally have been "intelligently guessed" and validated by numerous experiments. In addition, it was desirable to obtain an agreement between current and voltage waveforms obtained experimentally and calculated numerically from the model. However, according to the discussion in Sec. II it is obvious that a straightforward application of original similarity parameters P_1 and P_2 in our simplified model could lead to large errors comparing to experimental results. In this section it will be shown that a certain modification in similarity parameters allows us to receive a satisfactory agreement of numerical solutions with the experimental results. An analysis of a series of experiments closely satisfying the aperiodic condition of the discharge shows that there exists a unique correspondence between the original and the modified similarity parameters.

Equation (16) describes the discharge which occurs through the wire and water simultaneously. Here we introduce additional modification which formally accounts for more complex processes concerning the dynamics of the plasma channel formation, especially at the moment of wire explosion, where a nonautonomous formation of plasma channel occurs. We suppose that these processes slow down the decrease of wire conductivity near its explosion time and simultaneously decrease the rate of plasma channel conductivity growth due to residual conductivity of wire material leftover. This process can be accounted for by connecting in series conductivities y^3/α ($\alpha \ll 1$ is a small parameter) of a second order of magnitude to each of the parallel channels. Mathematically it is equivalent to replacing the dimensionless conductivities y_m and y_p by their functions,

$$\tilde{y} = \frac{y^2}{y + \alpha/y}.$$

Here $\alpha \ll 1$ is a small number such that $\tilde{y}\,\widetilde{y \to 1}\,y$ while in another extreme $\tilde{y}\,\widetilde{y \to \alpha}\,\alpha^2$. This means that this modification has an influence only on small values of y. An analysis of Eqs. (16b) and (16c) shows that in this case y_m and y_p decrease and grow more slowly, respectively, in the vicinity of complete wire evaporation time. The value of the parameter α has been obtained by fitting numerical and experimental results. The best fit for a range of analyzed experiments was found to be $\alpha = 0.02$. Following these modifications, Eqs. (16b) and (16c) are transformed into equations,

$$\frac{dy_m}{d\tau} = -P_2\left(\frac{\tilde{y}_m}{\tilde{y}_m + P_4\tilde{y}_p}\right)^2\left(\frac{du}{d\tau}\right)^2, \tag{16b$'$}$$

$$\frac{dy_p}{d\tau} = \frac{P_4\tilde{y}_p}{\tilde{y}_m + P_4\tilde{y}_p}\left(\frac{du}{d\tau}\right)^2, \tag{16c$'$}$$

with initial conditions $y_m = 1$, $y_p = 0$.

In addition, in order to obtain a good agreement between the calculated and experimental current waveforms, modified values of the original parameters P_2 and P_4 defined by Eqs. (7) and (14) have been substituted into Eq. (16). In Tables I and II we present the values of the original P_2, P_4 and the values of the modified similarity parameters P_2^* and P_4^* which were found from the comparison of numerical calculations and experiments. Two types of the underwater discharge circuits with a total inductance of 420 nH have been analyzed: the circuits with 5 μF (Table I) and 10 μF (Table II) capacitors charged up to 30 kV [11]. The spark constant was assumed to be $A = 1.8 \times 10^5$ V^2 s/m^2 [6,10]. The modified similarity parameter P_4^* was found to be determined by the formula

$$P_4^* = 0.8 P_1^{9/8}/P_3, \tag{17}$$

TABLE I. Experimental and theoretical similarity parameters for the 5 μF, 30 kV discharge circuit with Cu-wire electrical underwater discharge.

L (m)	D (mm)	P_1	P_2	P_2^*	P_4	P_4^*
0.035	0.1	0.269	87.7	292	1.46	1.73
0.035	0.2	0.067	5.48	26.2	0.36	0.36
0.035	0.3	0.029	1.08	4.82	0.16	0.14
0.035	0.5	0.010	0.14	0.49	0.06	0.04
0.035	0.6	0.007	0.06	0.21	0.04	0.03
0.035	0.8	0.004	0.02		0.02	
0.06	0.1	0.461	87.7	302	1.46	1.85
0.06	0.2	0.115	5.48	35.5	0.36	0.39
0.06	0.3	0.051	1.08	7.34	0.16	0.15
0.06	0.5	0.018	0.14	0.80	0.06	0.05
0.06	0.6	0.012	0.06	0.35	0.04	0.03
0.06	0.8	0.007	0.02		0.02	
0.085	0.1	0.653	87.7	303	1.46	1.93
0.085	0.2	0.163	5.48	40.7	0.36	0.40
0.085	0.3	0.072	1.08	9.30	0.16	0.16
0.085	0.5	0.026	0.14	1.09	0.06	0.05
0.085	0.6	0.018	0.06	0.48	0.04	0.03
0.085	0.8	0.010	0.02		0.02	

which is very close to the P_4 definition presented in Sec. III. The modified similarity parameter P_2^* was found to be determined by the formula

$$P_2^* = 15 P_2^{2/3}[1 - \exp(-12 P_1)]. \tag{18}$$

A comparison between the experimentally and numerically calculated obtained current waveforms is presented in Fig. 1.

TABLE II. Experimental and theoretical similarity parameters for the 10 μF, 30 kV discharge circuit with Cu-wire electrical underwater discharge.

L (m)	D (mm)	P_1	P_2	P_2^*	P_4	P_4^*
0.035	0.1	0.380	248.0	586	2.46	2.72
0.035	0.2	0.095	15.50	63.4	0.61	0.57
0.035	0.3	0.042	3.062	12.5	0.27	0.22
0.035	0.5	0.015	0.396	1.35	0.10	0.07
0.035	0.6	0.010	0.191	0.59	0.07	0.04
0.035	0.8	0.005	0.060	0.15	0.04	
0.06	0.1	0.652	248.0	591	2.46	2.91
0.06	0.2	0.163	15.50	80.1	0.61	0.61
0.06	0.3	0.072	3.062	18.3	0.27	0.24
0.06	0.5	0.026	0.396	2.17	0.10	0.07
0.06	0.6	0.018	0.191	0.97	0.07	0.05
0.06	0.8	0.010	0.060	0.26	0.04	
0.085	0.1	0.924	248.0	592	2.46	3.04
0.085	0.2	0.231	15.50	87.4	0.61	0.64
0.085	0.3	0.102	3.062	22.4	0.27	0.25
0.085	0.5	0.036	0.396	2.90	0.10	0.08
0.085	0.6	0.025	0.191	1.32	0.07	0.05
0.085	0.8	0.014	0.060	0.36	0.04	

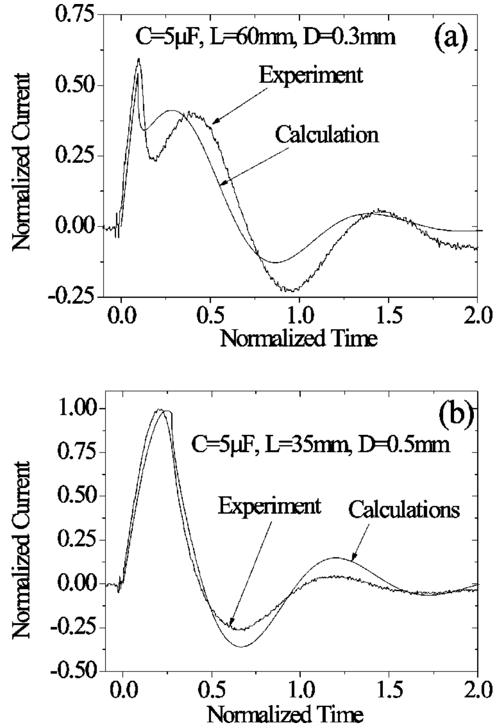

FIG. 1. Typical waveforms of the normalized discharge current obtained experimentally and from calculations for different cases of underwater electrical discharge with exploding Cu wire. Electrical circuit inductance 420 nH.

One can see a satisfactory agreement between the experiment and calculation. However, as mentioned earlier, the straightforward application of similarity parameters would not give a sufficient agreement of the model with an experiment. Therefore, the question may arise why similarity parameters work experimentally. The reason for this is that one and only one value of modified P_2^* and P_4^* corresponds to any P_2 and P_4.

VI. CONCLUSION

A simplified model of the underwater electrical discharge with a wire was presented. This model is based on a self-consistent solution of differential equations of the electrical circuit with time varying resistances of the wire and plasma discharge channels. These equations were derived with two simple assumptions regarding the loss of wire conductivity due to its uniform evaporation by the Joule heating and the increase of plasma channel conductivity due to the absorbed energy. It was found that the original similarity parameters which have been initially "intelligently guessed" and validated by experiments appear naturally in these differential equations, thus demonstrating the reason of similarity of (UED) and pointing out the main processes that govern the discharge. Also, it was shown that a straightforward application of the original similarity parameters in the differential equations of the model does not give solutions that show a satisfactory agreement with experimentally observed results. This is explained by the simplicity of the model which does not account for many sophisticated phenomena involved in

UED. Nevertheless, the fact of an experimentally observed similarity of different discharges indicates that all these sophisticated phenomena are inherently determined by these original similarity parameters. Therefore, for an adequate description of the experimental data by the use of our simplified model, modified parameters that are functions of original similarity parameters should be used to obtain a good agreement with the experimental data.

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